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Correction to "Dislocations in inhomogeneous media via a moduli perturbation approach: General formulation and two-dimensional solutions" by Yijun Du, Paul Segall, and Huajian Gao

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In the paper "Dislocations in inhomogeneous media via a moduli perturbation approach: General formulation and two- dimensional solutions" by Yijun Du, Paul Segall, and Huajian Gao *(Journal of Geophysical Research, 99*(B7), 13,767-13779, 1994), there are several substantive and typographical errors. Using a Fourier integral method, *Savage* [1998] recently found exact solutions for the surface displacement field from a finite edge dislocation (a model for a dip-slip fault) in a layered half-space. While some of his results show reasonable agreement with the perturbation solutions given in Du et al., several results disagree significantly. The disagreements are well beyond what could be ascribed to the approximate nature of the perturbation solutions.

The expressions given in the original Appendix B for the plane-strain displacement Green's functions from a line force in a two-dimensional, homogeneous half-space contain several sign errors, probably the result of an incorrect coordinate system shift, which are the source of the discrepancy between Du et al. and *Savage* [1998]. Additionally, one of the Green's functions (the vertical displacements from a vertically oriented line force) contains an error that can be traced back to *Maruyama* [1966] in this incorrect form. However, because the perturbation approach requires evaluation of the Green's functions only at the free surface, this error did not affect the analysis of Du et al.; the incorrect terms vanished at the free surface.

The expressions given in the original Appendix C for the stresses from an infinite, inclined edge dislocation in a homogeneous half-space also contain typographical errors, but these did not occur in the computer programs written to compute the perturbation solutions. Finally, the expressions given in the original Appendix D for the surface displacements from an infinite inclined edge dislocation in a homogenous half-space contain a sign error, but, again, this error did not occur in the computer programs.

To find accurate expressions for the two-dimensional plane-strain Green's functions, we began with the stress functions for a line force given by *Melan* [1932]. We checked these expressions extensively, ensuring that they satisfied the equilibrium and compatibility equations and that the tractions vanished at the free surface. Strains were calculated from the stresses, and these were then integrated to find displacements. Finally, we differentiated the displacement functions to confirm the integration.

To verify the accuracy of the stress functions for an edge dislocation we checked that they satisfied the equilibrium and compatibility equations and that the vertical stresses vanished at the free surface. We also compared them to the three-dimensional stresses for a dislocation given by *Okada* [1992], which they asymptotically approached as the three-dimensional dislocation grew very long.

Given in the appendices of this paper are corrected expressions for (1) displacements from a line force in a two- dimensional, homogeneous elastic half-space (these replace expressions (B4)-(B7) of Du et al.); (2) the stresses from an infinite edge dislocation in a two-dimensional, homogeneous elastic half-space (these replace expressions (C1)-(C3) of Du et al.); (3) the displacements from an infinite edge dislocation in a two-dimensional, homogeneous elastic half-space (these replace expressions (D1) and (D2) in Du et al.); and (4) stresses from a line force in a two- dimensional, homogenous elastic half-space (these were not given originally, but we include them here for completeness); Note that we have adopted a different coordinate system from that of Du et al., which is consistent with the coordinate system used by *Okada* [1992].

Using the corrected Green's functions, we compared the first order perturbation solutions to the case described in Figure 5a of *Savage* [1998]. Figure 1a compares the perturbation solutions derived from the incorrect Green's functions given by Du et al. to both *Savage's* [1998] solution and a solution derived from a finite-element analysis (these plots correspond to the original Figure 8, but are plotted using Savage's conventions). Note the severe disagreement for the horizontal displacement. The perturbation solution for the vertical displacement is also in error (it overcorrects), but at least the sign is correct. Figure 1b depicts the perturbation solutions using the correct Green's func-

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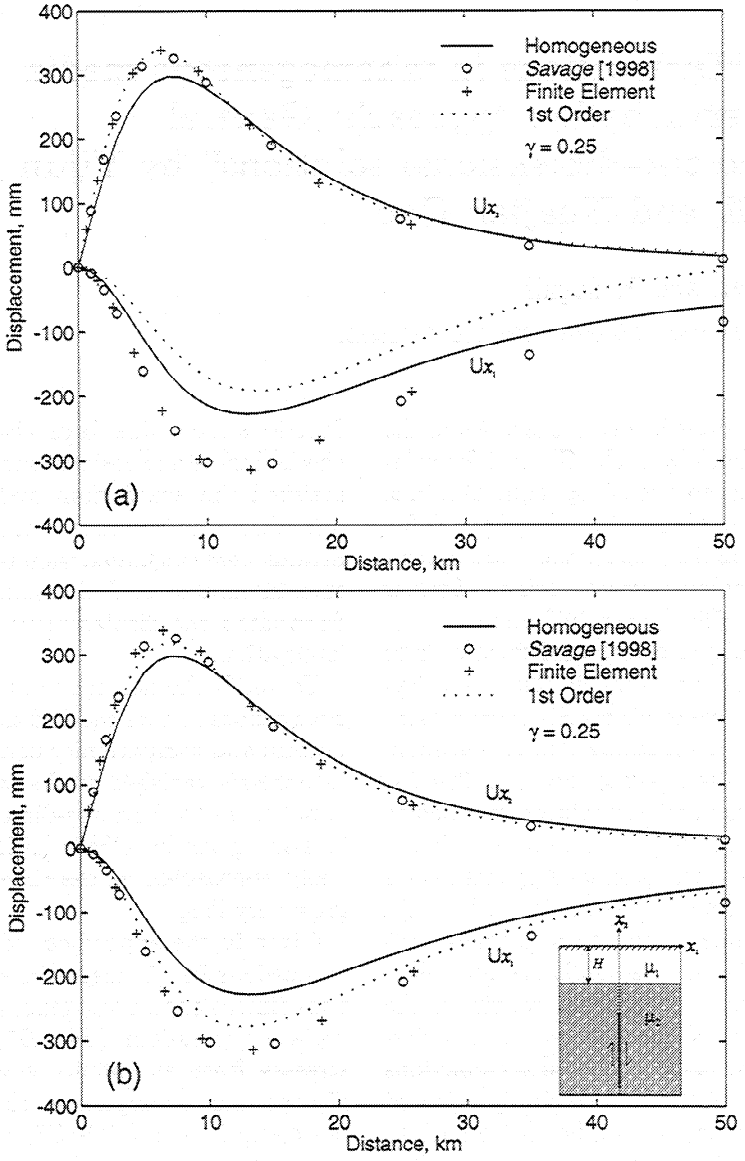

Figure 1. The first-order approximation calculated using (a) incorrect Green's functions and (b) correct Green's functions. Both are compared to a finite element solution and to *Savage's* [1998] Fourier intergral solution. The displacements are from a vertical fault in a layered, semi-infinite medium. The top layer is 5 km thick. The fault is 9 km deep, is 10 km long, is centered at the origin, and has 2 m slip. The x_1 axis is horizontal; the x_2 axis is vertical.

tions. The first order solutions for both displacement components now agree reasonably with the other solutions. Moreover, the first-order solution for the vertical component is now better behaved–it approaches rather than exceeds the analytic solution.

We have recreated the original Figures 8 and 9 using the correct Green's functions. While the new version (Figures 2 and 3) differ significantly from the originals, particularly for the horizontal displacements, the main geodetic conclusion remains the same: failure to account for material heterogeneity can at least partially explain the consistently shallow bias of geodetic inversion. Moreover, the good agreement among the perturbation solutions, the analytic solutions, and the finite element solutions validates the perturbation approach as a viable and efficient method for incorporating material heterogeneity into deformation models.

Appendix A: Definitions and Coordinate System

All the expressions given in these appendices assume the following definitions:

$$b_1 = s \sin\delta \tag{A1}$$

$$b_2 = s \cos\delta \tag{A2}$$

$$r^2 = (x_1 - \xi_1)^2 + \xi_2^2 \tag{A3}$$

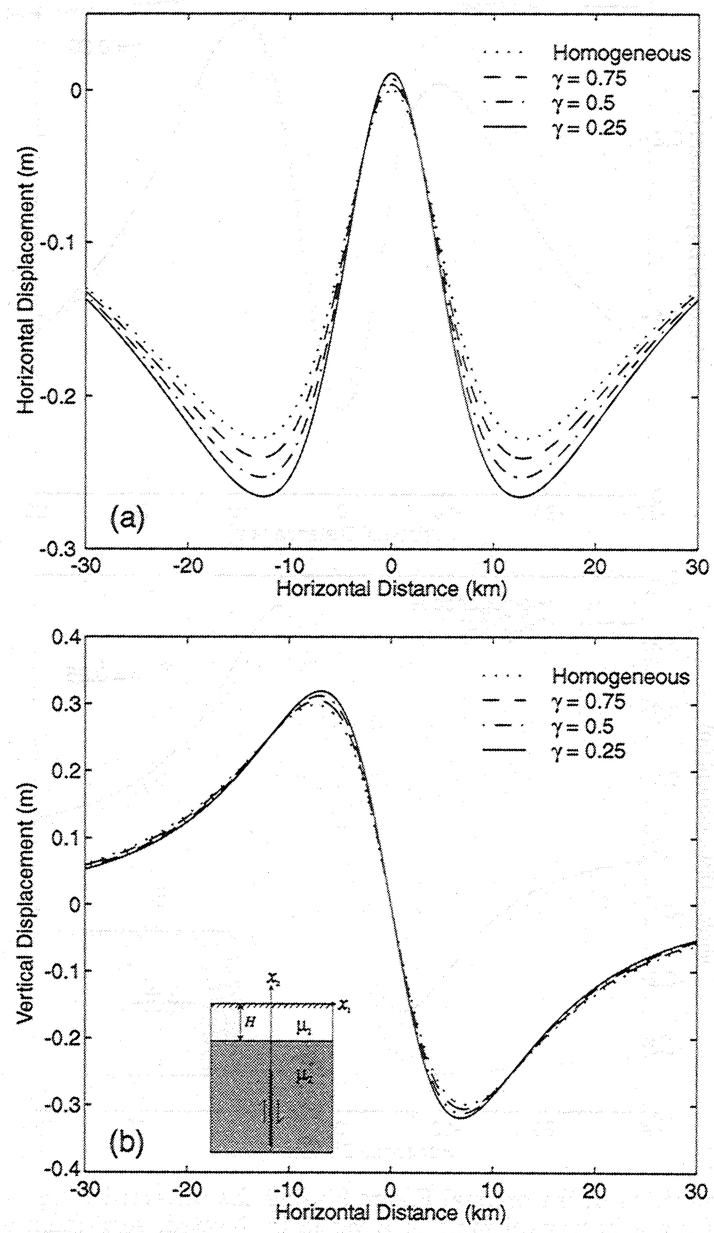

Figure 2. A reproduction of the original Figure 8 using the corrected Green's functions. The displacements are from a vertical fault in a layered, semi-infinite medium. The top, more compliant, layer is 5 km thick. The fault is 9 km deep, is 10 km long, is centered at the origin, and has 2 m slip.

$$r_1^2 = (x_1 - \xi_1)^2 + (x_2 - \xi_2)^2 \tag{A4}$$

$$r_2^2 = (x_1 - \xi_1)^2 + (x_2 + \xi_2)^2 \tag{A5}$$

$$\theta = \tan^{-1}\left[\frac{x_1 - \xi_1}{\xi_2}\right] \tag{A6}$$

$$\theta_2 = \tan^{-1}\left[\frac{x_1 - \xi_1}{x_2 + \xi_2}\right] \tag{A7}$$

where s is the slip vector and δ is the dip of the dislocation. The elastic moduli used in the following expressions are Poisson's ratio ν and the shear modulus μ.

Figure A1 depicts the coordinate system adopted here. Note that x_1 is the horizontal axis and that x_2 is the vertical axis, which is oriented positive up.

Appendix B: Displacements at (x_1, x_2) From a Line Force Acting at (ξ_1, ξ_2) in a Two-Dimensional Homogeneous Elastic Half-Space

For a line force in the x_1 direction,

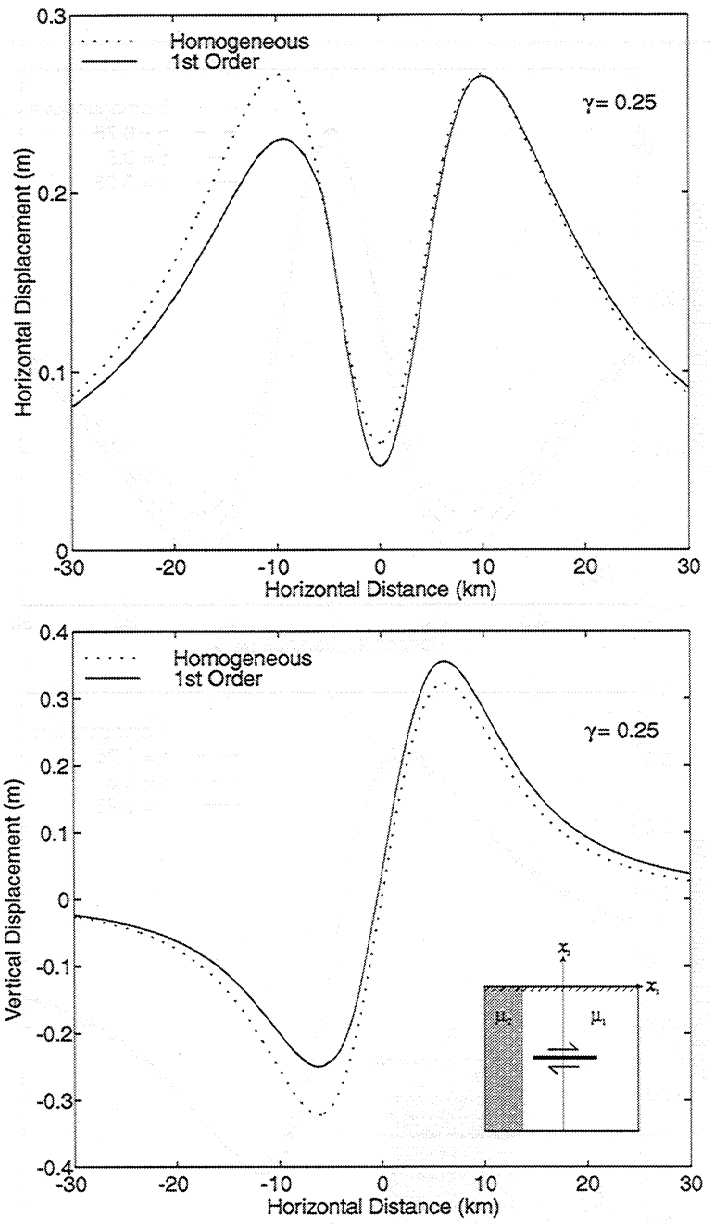

Figure 3. A reproduction of the original Figure 9 using the corrected Green's functions. The displacements are from a horizontal fault in a vertically layered, semi-infinite medium. The boundary between the layers is at −5.5 km; the more compliant medium is to the right. The fault is 9 km deep, is 8 km long, is centered at the origin, and has 2 m slip.

$$G_{11} = \frac{1}{2\pi\mu(1-\nu)}\left[-\frac{3-4\nu}{4}\ln r_1 - \frac{8\nu^2-12\nu+5}{4}\ln r_2 - \frac{(x_2-\xi_2)^2}{4r_1^2} - \frac{(3-4\nu)(x_2+\xi_2)^2+2\xi_2(x_2+\xi_2)-2\xi_2^2}{4r_2^2} + \frac{\xi_2 x_2 (x_2+\xi_2)^2}{r_2^4}\right] \quad \text{(B1)}$$

$$G_{11}\Big|_{x_2=0} = \frac{1}{2\pi\mu}\left[-\frac{\xi_2^2}{r^2} + (\nu-1)\ln r^2\right] \quad \text{(B2)}$$

$$G_{21} = \frac{1}{2\pi\mu(1-\nu)}\left[(1-2\nu)(1-\nu)\theta_2 + \frac{(x_2-\xi_2)(x_1-\xi_1)}{4r_1^2} + (3-4\nu)\frac{(x_2-\xi_2)(x_1-\xi_1)}{4r_2^2} - \frac{\xi_2 x_2 (x_2+\xi_2)(x_1-\xi_1)}{r_2^4}\right] \quad \text{(B3)}$$

$$G_{21}\Big|_{x_2=0} = \frac{1}{2\pi\mu}\left[-\frac{\xi_2(x_1-\xi_1)}{r^2} - (2\nu-1)\theta\right] \quad \text{(B4)}$$

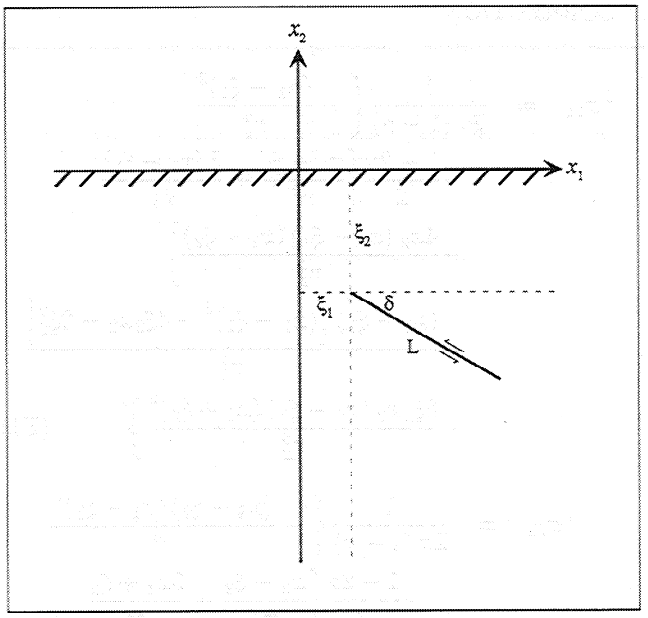

Figure A1. The coordinate system and dislocation geometry adopted here. Note that the vertical axis is negative down. The locations of both line forces and edge dislocations are denoted by (ξ_1, ξ_2), while the observation point is denoted by (x_1, x_2). The arrows indicate sense of motion for positive slip.

For a line force in the x_2 direction,

$$G_{12} = \frac{1}{2\pi\mu(1-\nu)}\left[-(1-2\nu)(1-\nu)\theta_2 + \frac{(x_2-\xi_2)(x_1-\xi_1)}{4r_1^2} + (3-4\nu)\frac{(x_2-\xi_2)(x_1-\xi_1)}{4r_2^2} + \frac{\xi_2 x_2 (x_2+\xi_2)(x_1-\xi_1)}{r_2^4}\right] \quad \text{(B5)}$$

$$G_{12}\Big|_{x_2=0} = \frac{1}{2\pi\mu}\left[-\frac{\xi_2(x_1-\xi_1)}{r^2} + (2\nu-1)\theta\right] \quad \text{(B6)}$$

$$G_{22} = \frac{1}{2\pi\mu(1-\nu)}\left[-\frac{3-4\nu}{4}\ln r_1 - \frac{8\nu^2-12\nu+5}{4}\ln r_2 - \frac{(x_1-\xi_1)^2}{4r_1^2} + \frac{2\xi_2 x_2 - (3-4\nu)(x_1-\xi_1)^2}{4r_2^2} - \frac{\xi_2 x_2 (x_1-\xi_1)^2}{r_2^4}\right] \quad \text{(B7)}$$

$$G_{22}\Big|_{x_2=0} = \frac{1}{2\pi\mu}\left[-\frac{(x_1-\xi_1)^2}{r^2} + (\nu-1)\ln r^2\right] \quad \text{(B8)}$$

Appendix C: Stresses at (x_1, x_2) From an Infinite Edge Dislocation at (ξ_1, ξ_2) in a Two-Dimensional Homogeneous Elastic Half-Space

$$\begin{aligned}\sigma_{11} = {} & \frac{\mu b_1}{2\pi(1-\nu)}\Bigg\{ + \frac{(x_1-\xi_1)\left[(x_2-\xi_2)^2-(x_1-\xi_1)^2\right]}{r_1^4} - \frac{(x_1-\xi_1)\left[(x_2+\xi_2)^2-(x_1-\xi_1)^2\right]}{r_2^4} \\ & + \frac{4\xi_2(x_1-\xi_1)}{r_2^6}\Big[(2\xi_2-x_2)(x_2+\xi_2)^2 + (3x_2+2\xi_2)(x_1-\xi_1)^2\Big]\Bigg\} \\ & + \frac{\mu b_2}{2\pi(1-\nu)}\Bigg\{ - \frac{(x_2-\xi_2)\left[(x_2-\xi_2)^2+3(x_1-\xi_1)^2\right]}{r_1^4} + \frac{(x_2+\xi_2)\left[(x_2+\xi_2)^2+3(x_1-\xi_1)^2\right]}{r_2^4} \\ & - \frac{2\xi_2}{r_2^6}\Big[6x_2(x_2+\xi_2)(x_1-\xi_1)^2 - (x_2-\xi_2)(x_2+\xi_2)^3 - (x_1-\xi_1)^4\Big]\Bigg\}\end{aligned} \quad \text{(C1)}$$

$$\begin{aligned}\sigma_{22} = {} & \frac{\mu b_1}{2\pi(1-\nu)}\Bigg\{ - \frac{(x_1-\xi_1)\left[3(x_2-\xi_2)^2+(x_1-\xi_1)^2\right]}{r_1^4} + \frac{(x_1-\xi_1)\left[3(x_2+\xi_2)^2+(x_1-\xi_1)^2\right]}{r_2^4} \\ & + \frac{4\xi_2 x_2(x_1-\xi_1)}{r_2^6}\Big[3(x_2+\xi_2)^2 - (x_1-\xi_1)^2\Big]\Bigg\} \\ & + \frac{\mu b_2}{2\pi(1-\nu)}\Bigg\{ - \frac{(x_2-\xi_2)\left[(x_2-\xi_2)^2-(x_1-\xi_1)^2\right]}{r_1^4} + \frac{(x_2+\xi_2)\left[(x_2+\xi_2)^2-(x_1-\xi_1)^2\right]}{r_2^4} \\ & + \frac{2\xi_2}{r_2^6}\Big[6x_2(x_2+\xi_2)(x_1-\xi_1)^2 - (3x_2+\xi_2)(x_2+\xi_2)^3 + (x_1-\xi_1)^4\Big]\Bigg\}\end{aligned} \quad \text{(C2)}$$

$$\sigma_{12} = \frac{\mu b_1}{2\pi(1-\nu)}\left\{ + \frac{(x_2-\xi_2)\left[(x_2-\xi_2)^2-(x_1-\xi_1)^2\right]}{r_1^4} - \frac{(x_2+\xi_2)\left[(x_2+\xi_2)^2-(x_1-\xi_1)^2\right]}{r_2^4} + \frac{2\xi_2}{r_2^6}\left[6x_2(x_2+\xi_2)(x_1-\xi_1)^2 - (x_1-\xi_1)^4 + (\xi_2-x_2)(x_2+\xi_2)^3\right]\right\} + \frac{\mu b_2}{2\pi(1-\nu)}\left\{ - \frac{(x_1-\xi_1)\left[(x_2-\xi_2)^2-(x_1-\xi_1)^2\right]}{r_1^4} + \frac{(x_1-\xi_1)\left[(x_2+\xi_2)^2-(x_1-\xi_1)^2\right]}{r_2^4} - \frac{4\xi_2 x_2(x_1-\xi_1)}{r_2^6}\left[3(x_2+\xi_2)^2 - (x_1-\xi_1)^2\right]\right\} \quad \text{(C3)}$$

To calculate the stresses from a finite edge dislocation, sum the stresses from two appriopriately juxtaposed infinite edge dislocations.

Appendix D: Surface Displacements at $(x_1, 0)$ From an Infinite Edge Dislocation at (ξ_1, ξ_2) in a Two-Dimensional Homogeneous Elastic Half-Space

$$u_1 = \frac{1}{\pi}\left[b_2\theta - \frac{b_1 + b_2\tan\theta}{1+\tan^2\theta}\right] \quad \text{(D1)}$$

$$u_2 = \frac{1}{\pi}\left[-b_1\theta + \frac{b_2 - b_1\tan\theta}{1+\tan^2\theta}\right] \quad \text{(D2)}$$

To calculate the surface displacements from a finite edge dislocation, sum the displacements from two appropriately juxtaposed infinite edge dislocations.

Appendix E: Stresses at (x_1, x_2) From a Line Force Acting at (ξ_1, ξ_2) in a Two-Dimensional Homogeneous Elastic Half-Space

For a line force in the x_1 direction,

$$^1\sigma_{11} = \frac{1}{2\pi(1-\nu)}\left\{ - \frac{(x_1-\xi_1)^3}{r_1^4} - \frac{1-2\nu}{2}\left[\frac{x_1-\xi_1}{r_1^2} + \frac{3(x_1-\xi_1)}{r_2^2} - \frac{4x_2(x_1-\xi_1)(x_2+\xi_2)}{r_2^4}\right] - \frac{(x_1-\xi_1)\left[(x_1-\xi_1)^2 - 4\xi_2 x_2 - 2\xi_2^2\right]}{r_2^4} - \frac{8\xi_2 x_2(x_1-\xi_1)(x_2+\xi_2)^2}{r_2^6}\right\} \quad \text{(E1)}$$

$$^1\sigma_{12} = \frac{1}{2\pi(1-\nu)}\left\{ - \frac{(x_2-\xi_2)(x_1-\xi_1)^2}{r_1^4} - \frac{1-2\nu}{2}\left[\frac{x_2-\xi_2}{r_1^2} + \frac{3x_2+\xi_2}{r_2^2} - \frac{4x_2(x_2+\xi_2)^2}{r_2^4}\right] - \frac{(x_2+\xi_2)\left[2\xi_2 x_2 + (x_1-\xi_1)^2\right]}{r_2^4} + \frac{8\xi_2 x_2(x_1-\xi_1)^2(x_2+\xi_2)}{r_2^6}\right\} \quad \text{(E2)}$$

$$^1\sigma_{22} = \frac{1}{2\pi(1-\nu)}\left\{ - \frac{(x_2-\xi_2)^2(x_1-\xi_1)}{r_1^4} + \frac{1-2\nu}{2}\left[\frac{x_1-\xi_1}{r_1^2} - \frac{x_1-\xi_1}{r_2^2} - \frac{4x_2(x_1-\xi_1)(x_2+\xi_2)}{r_2^4}\right] + \frac{(x_1-\xi_1)\left[\xi_2^2 - x_2^2 + 6\xi_2 x_2\right]}{r_2^4} - \frac{8\xi_2 x_2(x_1-\xi_1)^3}{r_2^6}\right\} \quad \text{(E3)}$$

For a line force in the x_2 direction,

$$^2\sigma_{11} = \frac{1}{2\pi(1-\nu)}\left\{ - \frac{(x_2-\xi_2)(x_1-\xi_1)^2}{r_1^4} - \frac{1-2\nu}{2}\left[-\frac{x_2-\xi_2}{r_1^2} + \frac{3\xi_2+x_2}{r_2^2} + \frac{4x_2(x_1-\xi_1)^2}{r_2^4}\right] - \frac{(x_2+\xi_2)\left[(x_1-\xi_1)^2 + 2\xi_2^2\right]}{r_2^4} + \frac{2\xi_2(x_1-\xi_1)^2}{r_2^4} - \frac{8\xi_2 x_2(x_2+\xi_2)(x_1-\xi_1)^2}{r_2^6}\right\} \quad \text{(E4)}$$

$$
\begin{aligned}
{}^{2}\sigma_{12} = \frac{x_1-\xi_1}{2\pi(1-\nu)}\Bigg\{ & -\frac{(x_2-\xi_2)^2}{r_1^4} \\
& -\frac{1-2\nu}{2}\left[\frac{1}{r_1^2}-\frac{1}{r_2^2}+\frac{4x_2(x_2+\xi_2)}{r_2^4}\right] \\
& -\frac{x_2^2-2\xi_2x_2-\xi_2^2}{r_2^4} \\
& -\frac{8\xi_2x_2(x_2+\xi_2)^2}{r_2^6}\Bigg\} \qquad \text{(E5)}
\end{aligned}
$$

$$
\begin{aligned}
{}^{2}\sigma_{22} = \frac{1}{2\pi(1-\nu)}\Bigg\{ & -\frac{(x_2-\xi_2)^3}{r_1^4} \\
& -\frac{1-2\nu}{2}\left[\frac{x_2-\xi_2}{r_1^2}+\frac{3x_2+\xi_2}{r_2^2}\right. \\
& \left.-\frac{4x_2(x_1-\xi_1)^2}{r_2^4}\right] \\
& -\frac{(x_2+\xi_2)\left[(x_2+\xi_2)^2+2\xi_2x_2\right]}{r_2^4} \\
& +\frac{8\xi_2x_2(x_2+\xi_2)(x_1-\xi_1)^2}{r_2^6}\Bigg\} \qquad \text{(E6)}
\end{aligned}
$$

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